# Geometry: One "G" in Masonry 

Presentation to the<br>New Jersey Lodge of Research \& Education<br>March 27, 2004<br>Trenton, New Jersey<br>Edgar M. Coster, PM<br>Madison Lodge No. 93, F. \& A.M.

## Introduction

By the time a man is raised to the sublime degree of Master Mason he has heard the "G" explained several times in our ritual and lectures. One sometimes wonders if the message is not lost in the virtual flood of information with which we inundate our candidates. Even those of us who perform the ritual may not have paused to reflect on the significance of geometry in every day life, and why it represents such a central role in our traditions and symbolism.

This work is by no means an exhaustive treatise on the subject of geometry. It is a modest attempt to demonstrate the relevance of geometry in a secular sense, and thereby understand the enduring importance of this favorite science of ours. Perhaps we can come to a greater appreciation of the wisdom of the founders of the Craft and the authors of our ritual. Masons are known to find the most solemn truths in the midst of the most innocent social pleasures. Using some props and practical demonstrations, we may even have a little fun in the process of this demonstration.

## Why geometry?

One of the core elements that we understand about Freemasonry is that it evolved in some way from the guilds of operative masons. Volumes have been written about our connections to the temple builders of the Bible and to their professional descendants who erected the great castles and cathedrals that so define the landscape and the culture of Europe. We believe that the founders of our fraternity saw in the builders' secrets, a useful and informative device for passing along the "secrets" of our fraternity.

Geometry's origins lie in the earliest efforts of man to control his environment. Herodotus, the fifth century Roman historian, credits the Egyptians with inventing geometry for two practical necessities. After the periodic flooding of the Nile it was necessary to survey lands and establish boundaries. The pharaohs also needed to measure volumes, as it was essential to account for tribute paid in grain, oil and other commodities. Especially since round vessels were often used to store commodities, it was not a simple matter to calculate their volumes.

As early as 3100 B.C. there is evidence that geometry was used to survey land, construct buildings and measure storage vessels. According to historians, the Greeks eventually expanded the science to astronomy and to the field of optics. Here we will confine the discussion to what is known as plane geometry, a part of Euclidian geometry. It is what many of us learned in "tenth grade math."

It is convenient to begin is with our ritual definition.
"Geometry treats of magnitudes in general where length, breadth and thickness are considered, from a point to a line, from a line to a superficies and from a superficies to a solid."

If we stretch our memories to our time in school when we first encountered Euclid, we may recall the basic building blocks that we today recite to our candidates. "A point is an indivisible part of space." It has no surface or size, although we conveniently indicate it on paper or on an illustration with a dot. Actually, it only represents a location.
"A line is a figure of one dimension, namely length." We were taught that a line connects two points in space. Like the point, the line really has no thickness or mass, but only a dimension. A straight line, of course, is the shortest distance between two points, and that is one of the earliest of the builder's secrets!

Let's complete the cast of characters with superficies and solids. "A superficies is a figure of two dimensions, namely length and breadth." One of the most often mispronounced words in the ritual is "superficies." Who has not heard giggles on the sidelines when the sound of this tongue twister suggests something to do with feces? It has a common root with the word superficial, and refers to the outward appearance or surface of a structure or thing. Not usually found in geometry texts today, Preston probably included it because it was a usage of the language of the late $18^{\text {th }}$ century.

Most commonly we think of squares and rectangles as superficies, but all two-dimensional shapes are, in fact superficies: (Illustration1)

## Illustration 1: Superficies



Another important consideration about a superficies is that it defines a plane. Actually, only three points are required. Two points give us one dimension; a third creates the plane on which we observe the superficies. This is easily illustrated by a draftsman's triangle. It represents three points on an invisible plane. We can rotate it on any axis and it will describe a plane in space. Fourth and subsequent points off the plane merely provide stability, such as we use for tables and similar objects. Lastly, "a solid is a figure of three dimensions, namely length, breadth and thickness."

We also note that,
"By this science the architect is enabled to construct his plans and execute his designs; the general to arrange his soldiers; the engineer to mark out ground for encampments; the geographer to give a description of the world delineate the extent of seas; and specify the divisions of empires, kingdoms and provinces."

But what has that to do with us? Why, as Masons do we need to know this, other than that knowledge is something our fraternity values for its own sake? When we examine how the architect's designs are put to use, we begin to see the first glimmer of light.

To build any basic structure certain fundamentals must be observed: the foundation must be level, so that all horizontal surfaces in the structure will be as it rises from the ground. The walls and vertical parts must be plumb for stability and strength. The corners and joints must be square and true, or the structure will not stand. One famous structure illustrates what a structure looks like when its foundation is not level. In this case, the tower was built on ground that later subsided. The later attempts to re-level it are apparent.

## Illustration 2: The Leaning Tower of Pisa



For centuries Masons have learned that even the simplest structure must be level, plumb and square. Where have we heard that trinity uttered before? And what better way to illustrate how a man's character should be built?

## The Secrets of Early Builders

We will see that builders' secrets made possible all of the constructions that marked the earliest civilizations after man emerged from the natural protection of caves. Dwellings, storage
buildings, barns and warehouses were no doubt first. Later came temples, churches and the great cathedrals. At the same time there evolved fortifications, ports and castles, arenas, theatres and stadiums. All the more remarkable is the fact that thousands of laborers and skilled craftsmen were needed to construct these larger, civic works in centuries during which few men could read or write. The essential skills had to be passed down from man to man, from mouth to ear and by illustrations drawn in sand or committed to a more permanent medium such as a trestle board. We will review four principle secrets of operative masonry that are reflected in our gentle Craft: The Square Foundation, the Pythagorean theorem, Measurement and How to Construct Working Tools.

## Secret \# 1: The Square Foundation

We can illustrate the basic squaring of a foundation with the following rules and tools. We start with a beginning point and the first length, which enables us to locate the second point. Using a rope or string, we determine the shortest distance between the two, and thus establish a straight line that becomes the base of the foundation. (Illustration 3, line 1,2). So far so good. Child's play. But how do we make sure the line that will define the side of the building is not only straight, but also at a right angle to the base line? Here we can use the simplest "compass" consisting of another line or string. With that line we can define a radius of a circle and use the circle to describe arcs that will help us to define a perpendicular to the base line. (3a and 3b.)

## Illustration 3: Defining a Perpendicular



We repeat the step with another circle in such a way that the two intersect each other both above and below the base line. If we then carefully draw a line connecting the two points of
intersection, (4) we have bisected the baseline and created a perpendicular to it. Where the lines intersect, we have four perfect right angles. There is a complex mathematical explanation as to why this is so but, to the craftsman, the theory is less important than the result of its application. Using any of these angles we have determined the starting corner of the building.

The next step is to measure out the distance along the side, extending the line beyond the measurement so that another perpendicular can be constructed to define another right angle and we can proceed to continue until the square or rectangle is laid out. The next "secret" is how to try the squareness of the foundation lines by measuring diagonals. This method is still used today when builders lay out something as simple as the slab foundation for a backyard shed, or a more complex foundation consisting of multiple shapes.

Illustration 4 shows a rectangle representing a marked out foundation. If we measure from A to C and from $B$ to $D$, the dimensions of both diagonal lines will be equal to each other. If the figure is square, triangle CBA and triangle CDA share a side, AC. (Looked at the other way, CDB and DAB also share a side, DB. This method helps builders assure themselves that they have constructed proper right angles when marking out the dimensions of all sides of the figure. It is a part of the "secret" of squaring foundations that is as old as the earliest buildings.

## Illustration 4: Trying a Square Foundation with Diagonals



> | If $\mathrm{AC}=\mathrm{BD}$ |
| :--- |
| The |
| foundation |
| is square. |

## Secret \#2: The Pythagorean Theorem

We have demonstrated why right angles have such practical importance. The earliest Western scholars sought to find the right expression of the relationship among the sides of the right angle triangle. In his Forty-seventh Problem, Euclid sought a foolproof method of defining it mathematically. The problem was simply stated yet it proved very difficult to solve because it is a rare right angle triangle whose sides are all whole numbers.

Legend has it that Pythagoras, when discovering the answer, cried out, "Eureka!" I have found it! Today's references question the veracity of this account. According to the Encyclopedia Britannica, it is likely that mathematicians in Pythagoras' school developed the theorem many years after his death. Whatever it's provenance, the theorem attributed to him has inestimable value:
"The square on the hypotenuse is equal to the sum of the squares on the opposing sides."

## Illustration 5: The Pythagorean Theorem



This is most often illustrated with the symbol we are all familiar with. Look at the officer's collars in New Jersey, and elsewhere, and you will see it. The formula is also often demonstrated by the use of the 3-4-5 triangle, because the arithmetic is simple. Three squared is 9 ; four squared is sixteen. Together they add up to twenty-five. Five squared is, of course, twenty-five.

The discovery of the mathematical formula may actually have been derived by backing into it from the observation of an ancient solution to the right angle problem. Egyptian builders were known as "rope pullers" because they used a knotted rope to lay out constructions and right angles. The rope was twelve units in length. (Note, it matters not whether the unit is inch, foot, cubit, or whatever.) One knot was place three units from one end and another five units from the other end. This left four units between the two. When drawn tight, with the ends joined and the two knots held tight, the rope forms a right-angle triangle, and a true corner can be described.

India's Vedic scriptures include what are known as the "Sulvasutra", or rules of the rope. Included among these rules is one used for laying out altars. It is a triad of 3-4-5 dimensions, identical to the one ascribed to the Egyptians. Babylonian tablets suggest triads in use 1,000 years before Euclid lived. How interesting that 3-4-5 appears in so many distant cultures. It is a simple but powerful truth with universal application. How appropriate to Freemasonry!

## Secret \# 3 - Reliable and Consistent Measurement

Another element of the builders' secrets was how to measure, and how to ensure that the measure was the same each time. Consider the use of body measurements: for an inch, a finger joint; for a foot of linear measure, a person's foot; for a yard, the distance from tip of nose to tip of finger with arm extended. The system was good news and bad news.

The good news is that, once instructed, a master workman could use his own, built-in measures to construct tools and measuring instruments. As long as the measures were consistent, it did not matter whether an inch represented 2.4 or 2.5 centimeters, or some other equivalent. The bad
news is that, if there were multiple craftsmen working on a structure, they had to take care not to use inconsistent measures. Obviously, when the builder's art progressed in sophistication, standard measures were adopted. A master would travel with proven tools, from which others could be constructed, such as a measuring rod or chain, which would have sufficient rigidity to avoid the stretching that, might affect a rope or leather strap. In the earliest days, however, a master mason was always equipped to measure and lay out his work because of his built-in measurements.

## Secret \# 4 - Constructing Working Tools

The early builders, of course, worked mostly in stone. Blocks of stone were cut in quarries to standard lengths and proportions. Mud bricks were made in forms of uniform dimensions so that the courses of brick would be uniform and sections could be divided easily. The operative mason used working tools we can list in the familiar 3-5-7 pattern. The core three are the plumb, level and square; the next two are the 24 -inch gauge and the compasses. He also used the gavel and the trowel which while important were less precise and more easily fashioned.

The twenty-four inch gauge is still used by bricklayers today. The use of 24 inches is interesting. As we all know, the number twelve is extremely helpful in measuring because it can be divided evenly by $2,3,4$ and 6 . By using twenty-four inches, we also can divide the measure by 8 and by 12. This tool, like the modern one here, also folds up neatly for easy storage and transportation.

The key to properly laying a course of stone or brick, and hence a wall, is to keep each course level and plumb, and to square corners and joints. Once properly trained, a master mason no doubt could construct his own tools. It all starts with making a true square. The square can then be used to make a plumb, and if you set a plumb on a straight edge, you have a level. But how to first make the square, "a right angle, horizontal and perpendicular?" That's it: a right angle, horizontal and perpendicular.

Using a divider or compasses in the same manner as we laid out a foundation on the ground, it is possible to describe a straight line and construct a perpendicular to it. Further, knowing the secret of the 3-4-5 triad, masons could easily try the square and other tools they used. The earliest working tools were probably made out of olive wood or a similarly dense hard wood, which would not warp or deform with dampness, dryness or exposure to heat or cold. Later, tools were made of cast metal to insure their durability and precision. Medieval guild members of all trades and crafts prized their proven working tools that could be used not only to insure quality and standards, but also to fashion additional tools and to test the tools carried by other workmen.

## Illustration 6: The Plumb and Level



Before modern masons perfected the spirit level, the plumb was constructed by fashioning a piece of wood with parallel, straight sides, as in the picture on the left in illustration 6. A small plumb line string and bob were attached so that the string would be aligned with a reference line at the center of the plumb. When held against a vertical object the line would tell whether or not the object was plumb. As is evident in the picture of the level, on the right, it was really nothing more than a plumb set on a broader base. Like the plumb, it had a line and bob so that, when set upon a horizontal surface, the line would tell if that surface was or was not level.

## The Basic Secrets

We can conclude that our operative brethren had these four elemental secrets:

1. How to construct a square foundation
2. Pythagoras' Theorem, or the 3-4-5 triad
3. Reliable measurement
4. How to construct true working tools

There was also a fifth "secret" consisting of the knowledge that memory and demonstration were essential to passing on the craft. Since all but the ruling class and the religious leaders were illiterate, instruction could not be by written word, but only by word of mouth and demonstration. This also worked to preserve secrecy, as there were no written manuals or instructions that could be obtained by persons who had not been accepted into the guild and properly trained and tested.

## Simplicity

Recently, an anecdote has circulated on the World Wide Web about the ridiculous nature of bureaucracy. It is relevant to our discussion. The important Pythagorean theorem contains 24 words in English. The Lord's Prayer has 66. The Gettysburg address contains 286 and the Declaration of Independence, 1,300 . The US government regulations on the sale of cabbage contain 26,911 words.

Our operative brethren were able to instruct apprentices and fellows of the craft with a relatively simple set of principles and a mere handful of tools. The greatest edifices of civilization throughout Europe, Asia and even pre-Columbian America -- were constructed with little more than these basics. We, in speculative Masonry, have taken this approach as our own. While there are volumes of work on Freemasonry which can occupy us for decades of study, we always come back to the basic elements of our craft that have assisted so many in the erection of their "spiritual, moral and Masonic edifices."

## Conclusion

We allude to the "G" as representing "Geometry, the first and noblest of sciences on which the superstructure of freemasonry is erected." We instruct our brethren that,
"The attentive ear receives the sound from the instructive tongue, and the mysteries of freemasonry are safely lodged in the repository of faithful breasts. Tools and implements of architecture, and symbolic emblems most expressive, are selected by the
fraternity to imprint upon the mind wise and serious truths; and thus through a succession of ages, are transmitted, unimpaired, the most excellent tenets of our fraternity."

What are these wise and serious truths? They are the basic tenets of Freemasonry.
To be good and true is the first lesson we are taught in Masonry.
To walk uprightly in all our stations before God and man.
To square our actions by the square of virtue and morality.
To circumscribe our desires and keep our passions within due bounds with all mankind.
And so, when we conclude the meeting today and quit this sacred retreat to mix again with the world,

How should Masons meet? On the level. How should they act? By the plumb. And part upon the square.
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## Addendum

## Use of Props

This presentation was accomplished using the following "props" for demonstrating the discussion points. Their use is recommended to anyone who would share the study with a live audience since demonstration appeals to more of the senses than just to hearing. Props: Three ropes, one of about ten feet, one of $25 \%$ of the first length, to demonstrate the perpendicular. A rope of twelve feet with knots measured $3^{\prime}$ from one end and $5^{\prime}$ from the other, used to demonstrate the triad. To illustrate the plane, a large draftsman's triangle. To show modern equivalents of level and plumb in metal, a carpenter's square and spirit level. A modern 24-inch mason's gauge. If available, also use working tools and an officer's collar.

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